AUTOMATIC SEARCH OF RECTANGLE ATTACKS ON FEISTEL CIPHERS

Application to WARP

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Slides: 20

Table of contents

1 Differential Crytanalysis and Boomerang Attacks

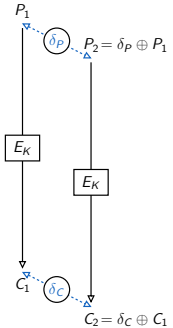
2 Automatic Search of Rectangle Attacks on WARP

3 Outlooks and Conclusion

Differential Crytanalysis and Boomerang Attacks

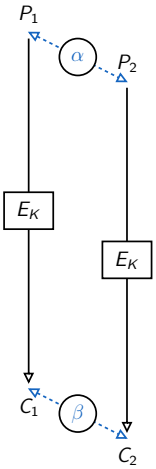
Differential Cryptanalysis [BS91]

- Symmetric ciphers
- Differential Cryptanalysis



$$\mathbb{P}(\delta_P \rightsquigarrow \delta_C)?$$

Boomerang [Wag99]



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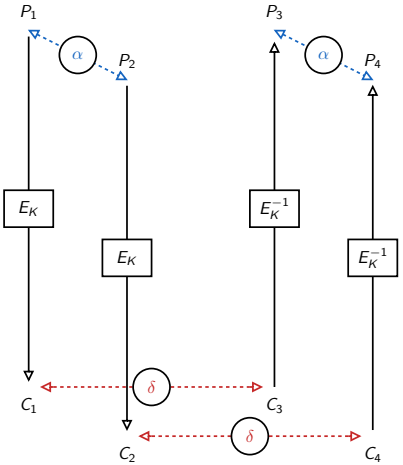

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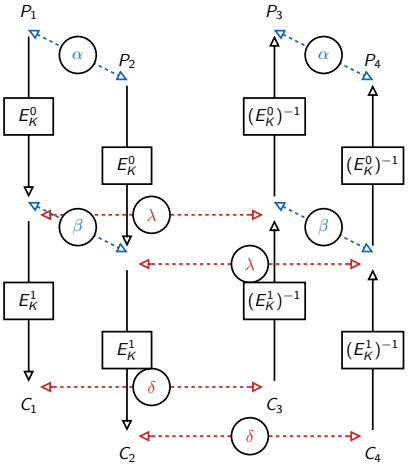
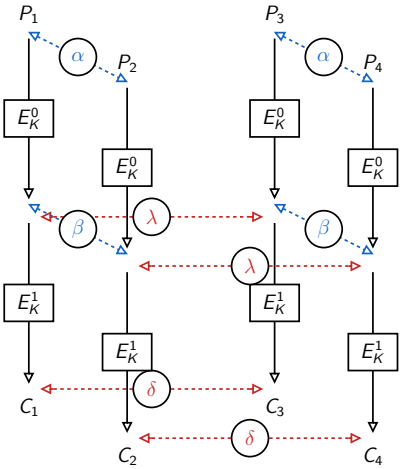


Rectangle Attack [BDK01]



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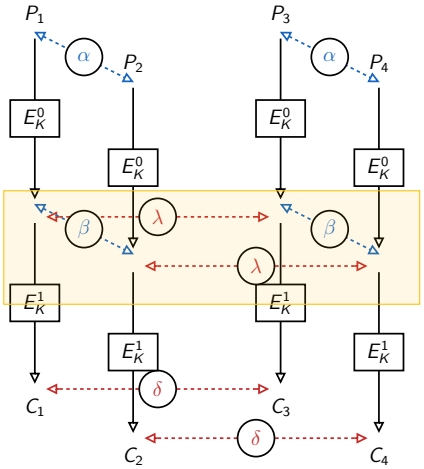
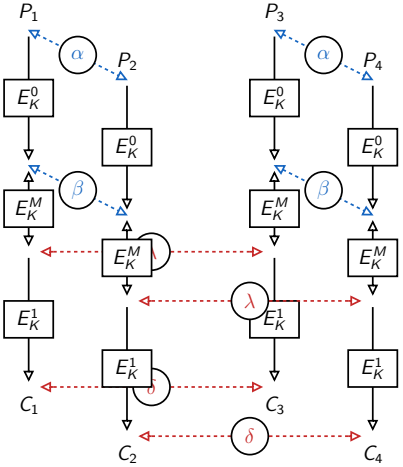


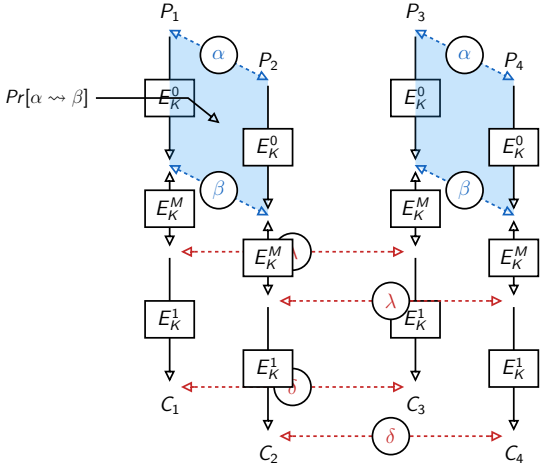
Are the two trails compatibles ?

Sometimes better [BK09]
Sometimes worst [Mur11]

Sandwich Attack [DKS10]



Sandwich Attack [DKS10]



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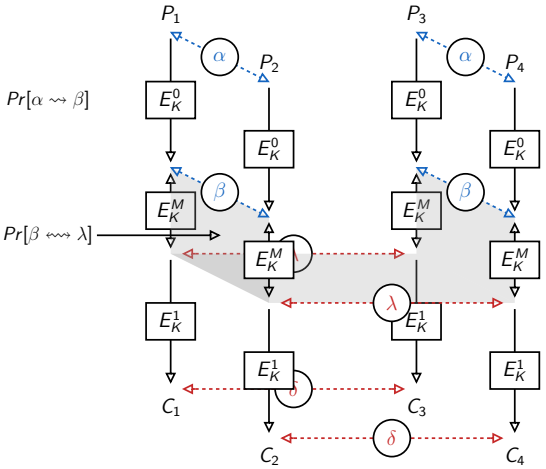

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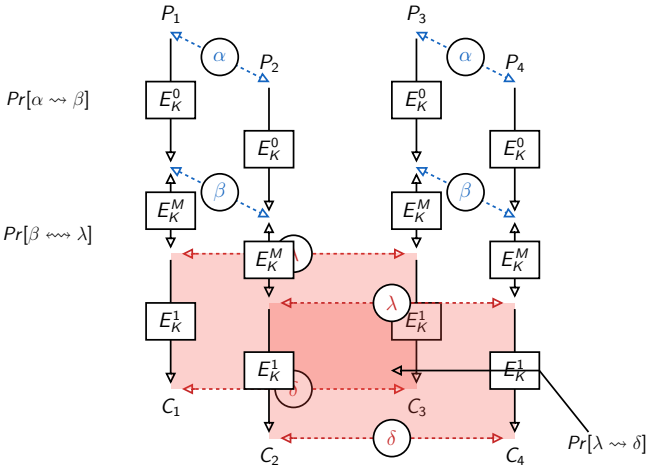

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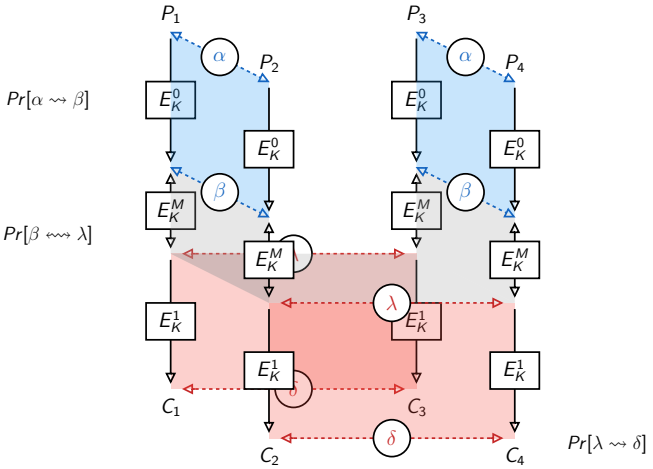
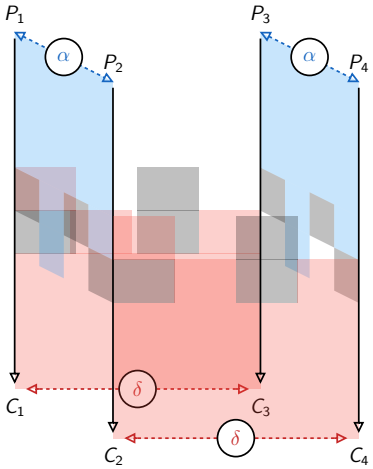
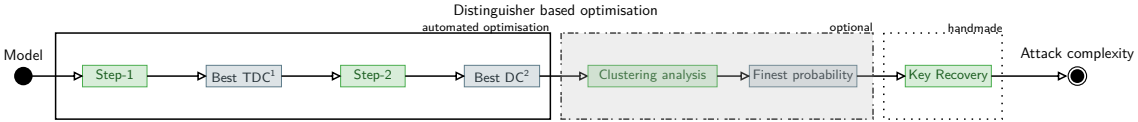


$$Pr[\alpha \leftrightsquigarrow \delta] = Pr[\alpha \rightsquigarrow \beta]^2 \times Pr[\beta \leftrightsquigarrow \lambda] \times Pr[\lambda \rightsquigarrow \delta]^2$$

Model of Delaune et al. [DDV20]

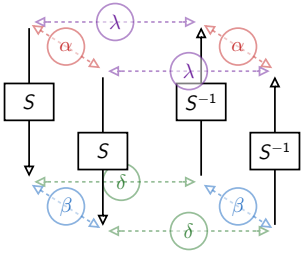


2 Steps (+KeyRecovery) solving process



[1]: Truncated Differential Characteristic
[2]: Differential Characteristic

Boomerang Transition in the Model of Delaune et al.



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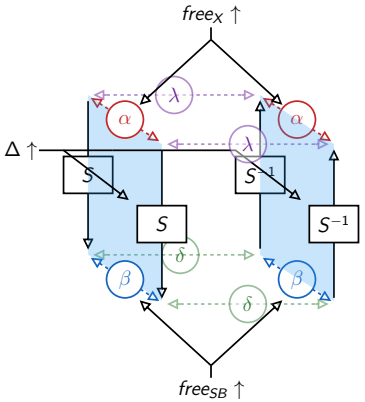

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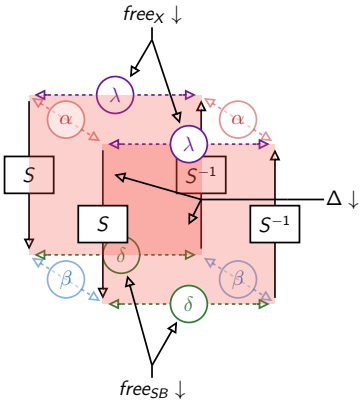
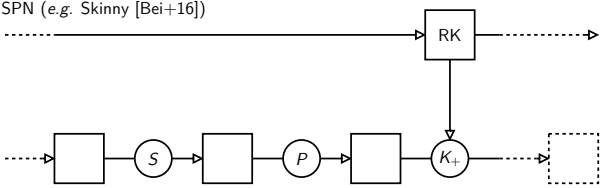


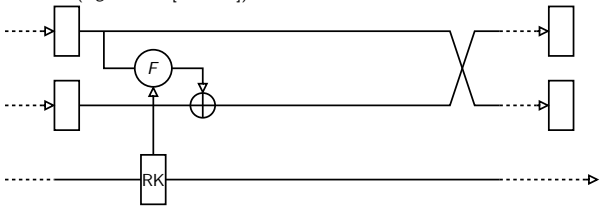
Motivation

Adapt the model of Delaune et al. to Feistel Networks

SPN (*e.g.* Skinny [Bei+16])



Feistel Network (*e.g.* WARP [Ban+20])



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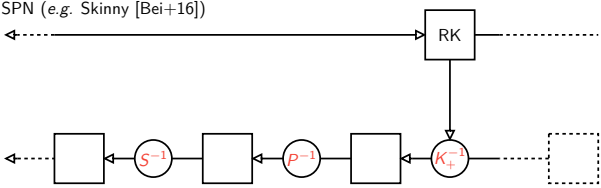
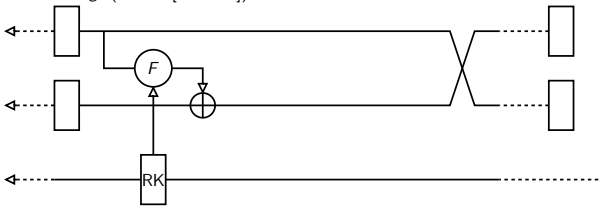
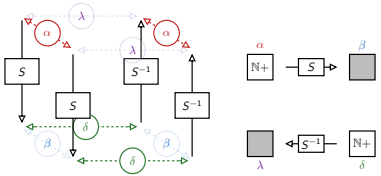


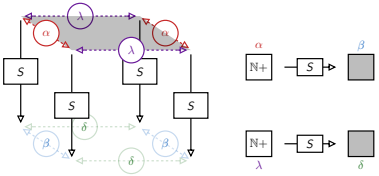
Feistel Network *e.g.* (WARP [Ban+20])



SPN vs Feistel Boomerang Transitions



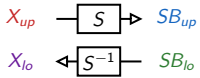
BCT [Cid+18]



FBCT [Bou+20]

S-Box Rules

Delaune et al.



- Rule 1

$$free_{Xup} \implies free_{SBup}$$
$$free_{SBlo} \implies free_{Xlo}$$

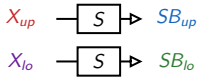
- Rule 2

$$free_{SBup} \implies \Delta_{Xup}$$
$$free_{Xlo} \implies \Delta_{Xlo}$$

- Rule 3

$$\neg free_{Xup} \vee \neg free_{Xlo}$$
$$\neg free_{SBup} \vee \neg free_{SBlo}$$

Feistel Adaptation



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- Rule 2

$$free_{SBup} \implies \Delta_{Xup}$$
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- Rule 3

$$\neg free_{Xup} \vee \neg free_{SBlo}$$
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Automatic Search of Rectangle Attacks on WARP

WARP

- Presented at SAC 2020 by Banik et al. [Ban+20]
- Compact hardware implementation
- 128-bit key and block (41 rounds)

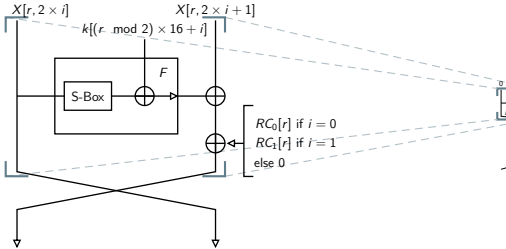


FIGURE 1 Close-up of two branches

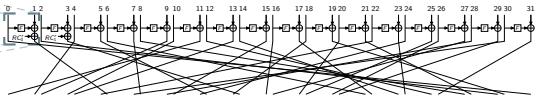


FIGURE 2 One round of WARP

Similarities with Delaune et al.'s model?

- The boomerang representation
- The search steps

Differences with Delaune et al.'s model?

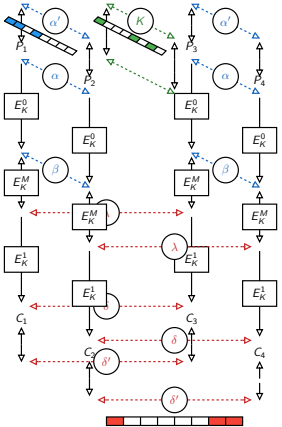
- Specific optimizations dedicated to WARP
- The S-Box representation
  - S-Box rules
  - Transition tables
- **Integration of the attack complexity in the optimisation process**

Key Recovery Automatisation

Attack of Zhao and co-authors [Zha+20]

$$2^{m_b+n/2} \cdot \sqrt{s} \cdot \frac{1}{\sqrt{p^2q^2r}} \cdot \frac{N_b}{N_b+N_d+N_f}$$

$$2^{m_b-n+2r_f}/(p^2q^2r)$$



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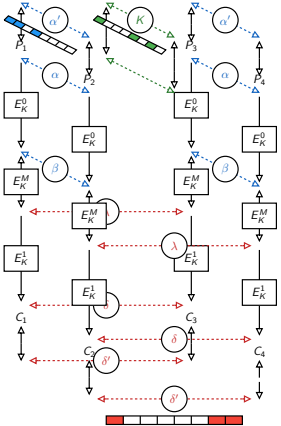

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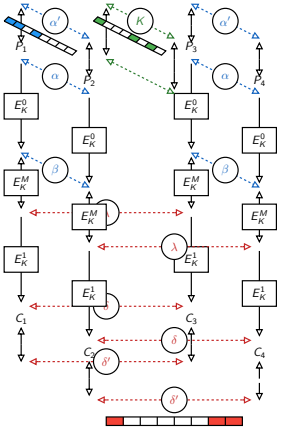

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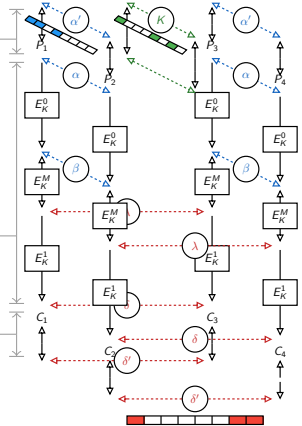

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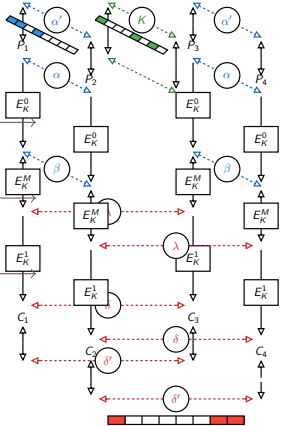
$$2^{m_b+n/2} \cdot \sqrt{s} \cdot \frac{1}{\sqrt{p^2q^2r}} \cdot \frac{N_b}{N_b+N_d+N_f}$$

$$2^{m_b-n+2r_f} / (p^2q^2r)$$



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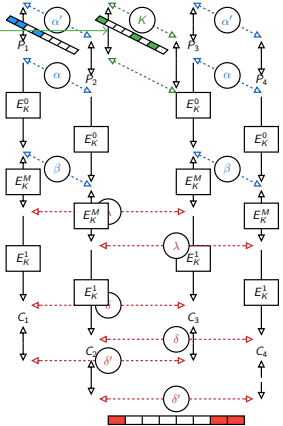

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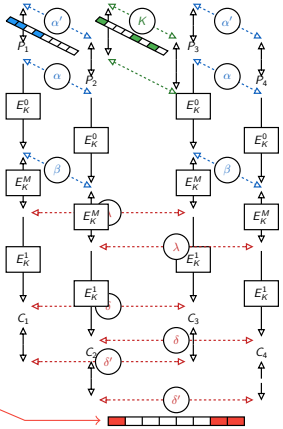

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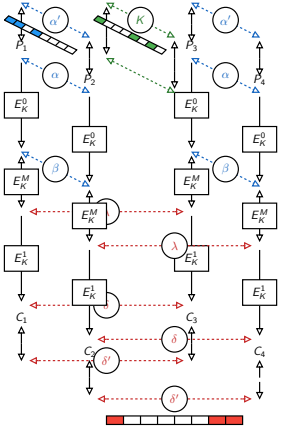
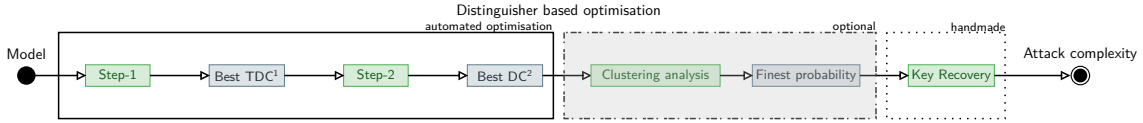
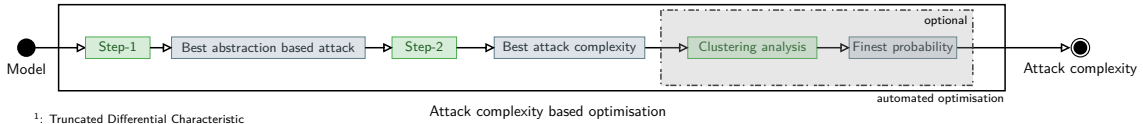
$$2^{m_b-n+2r_f}/(p^2q^2r)$$



Optimisation Process





[1]: Truncated Differential Characteristic
[2]: Differential Characteristic

Results on WARP

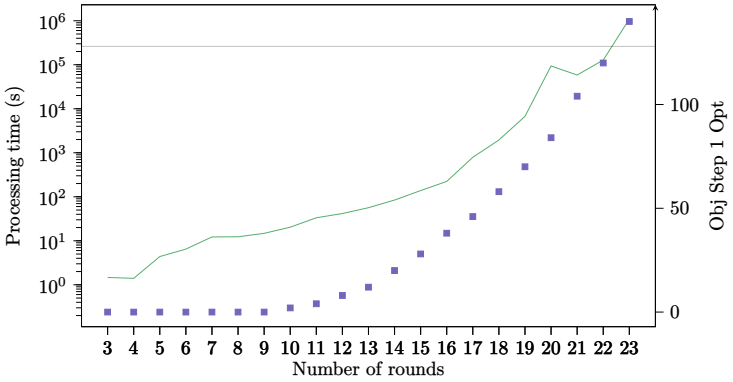


FIGURE 3

Execution time for Step-1 and Step-2 (—).
Best probability found with Step-1 Opt (■).
The black line corresponds to the probability 2^{-128}.

Experimental evaluation

Rounds	Model	Experiment	Number of tries
3	2^{0}	2^{0}	$2^{4} \times 16$
4	2^{0}	2^{0}	$2^{4} \times 16$
5	2^{0}	2^{0}	$2^{4} \times 16$
6	2^{0}	2^{0}	$2^{4} \times 16$
7	2^{0}	2^{0}	$2^{4} \times 16$
8	2^{0}	2^{0}	$2^{4} \times 16$
9	2^{0}	2^{0}	$2^{4} \times 16$
10	2^{-2}	$2^{-1.93}$	$2^{6} \times 16$
11	2^{-4}	$2^{-3.94}$	$2^{8} \times 16$
12	2^{-8}	$2^{-6.39}$	$2^{12} \times 16$
13	2^{-12}	$2^{-8.80}$	$2^{16} \times 16$
14	2^{-20}	$2^{-18.02}$	$2^{24} \times 16$
15	2^{-28}	$2^{-25.65}$	$2^{28} \times 16$
16	2^{-38}	$2^{-35.65}$	$2^{36} \times 11$

TABLE 1 The experimental evaluation of our Model on Warp.

The source code is available at:
`https://gitlab.inria.fr/lrouquet/boomerang-distinguisher-experimental-evaluation-on-WARP`

Results on WARP

Technique	Rounds	Probability	Time	Data	Mem.	Ref.
DC distinguisher	18	2^{-122}	-	-	-	[KY21]
DC distinguisher	20	$2^{-122.71}$	-	-	-	[TB22]
ID distinguisher	21	1	-	-	-	[Ban+20]
Boomerang distinguisher	21	$2^{-121.11}$	-	-	-	[TB22]
Boomerang distinguisher	23	2^{-124}	-	-	-	[This Work]
Boomerang distinguisher	23	$2^{-115.59}$	-	-	-	[HNE22]
Differential attack	21	-	2^{113}	2^{113}	2^{72}	[KY21]
Differential attack	23	-	$2^{106.68}$	$2^{106.62}$	$2^{106.62}$	[TB22]
Rectangle attack	24	-	$2^{125.18}$	$2^{126.06}$	$2^{127.06}$	[TB22]
Rectangle attack	26	-	$2^{115.9}$	$2^{120.6}$	$2^{120.6}$	[This Work]

Results on TWINE and LBlock-s

Cipher	Distinguishers	Rounds	Probability	Ref.
TWINE	Boomerang distinguisher	15	$2^{-58.92}$	[TB22]
TWINE	Boomerang Distinguisher + Clustering	15	$2^{-47.7}$	[This Work]
TWINE	Boomerang Distinguisher	15	$2^{-51.03}$	[HNE22]
TWINE	Boomerang distinguisher	16	$2^{-61.62}$	[TB22]
TWINE	Boomerang Distinguisher + Clustering	16	$2^{-59.8}$	[This Work]
TWINE	Boomerang Distinguisher	16	$2^{-58.04}$	[HNE22]
LBlock-s	Boomerang distinguisher	15	$2^{-58.64}$	[TB22]
LBlock-s	Boomerang Distinguisher + Clustering	16	$2^{-56.14}$	[Bou+20]
LBlock-s	Boomerang Distinguisher + Clustering	16	$2^{-54.8}$	[This Work]
LBlock-s	Boomerang Distinguisher	16	$2^{-53.59}$	[HNE22]

Outlooks and Conclusion

Summary

- We provide an automatic tool to search rectangle attacks on WARP
- The model can be easily adapted to other Feistel networks
- The model can be overtaken by the model of [HNE22] when the clustering effect is important
- We make a concession on distinguisher probabilities in favour of the lower attack complexities

Further search

- Integration of Boomerang and Rectangle attacks in Tagada [Lib+21]

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