

SoK: Functional Graphs and Their Applications in Generic Attacks on Iterated Hash Constructions

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Outline

Functional Graph

Preliminaries

Attacks on Hash-based MAC Based on FG

Attacks on Hash Combiners Based on FG

Summary and Open Problems

The Functional Graph of Random Mappings (FG)

Let $f \xleftarrow{\$} \mathcal{F}_N$. \mathcal{FG}_f is a directed graph, whose nodes are $0 \dots N - 1$ and edges are $\langle x, f(x) \rangle$

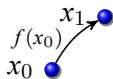


The Functional Graph of Random Mappings (FG)

x_0 ●

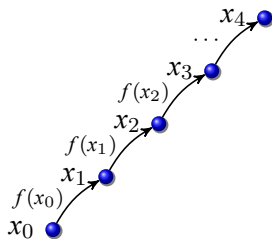


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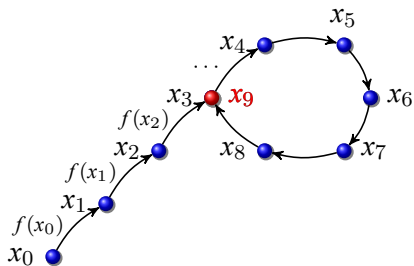


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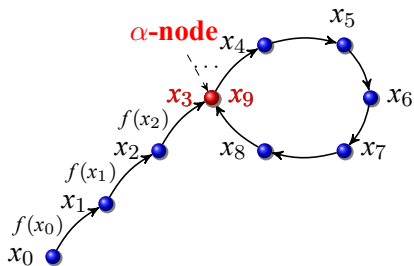


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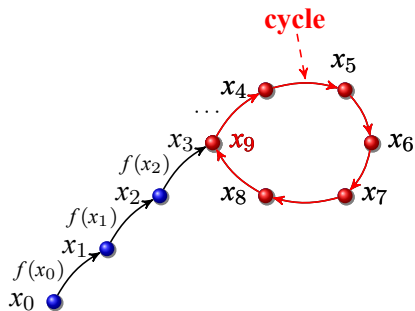


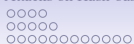
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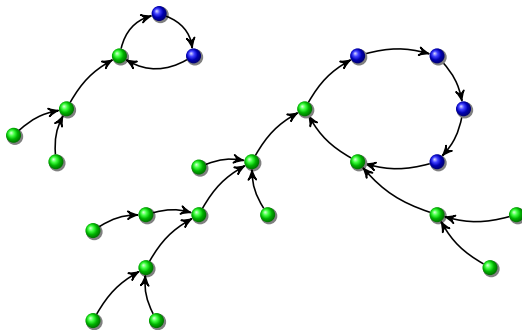


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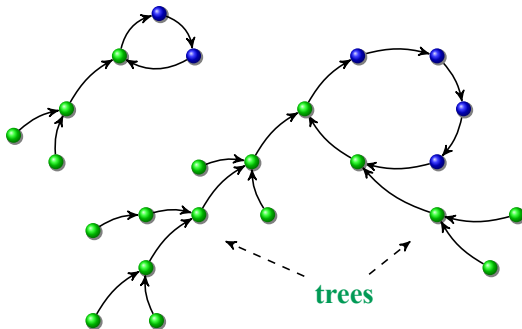


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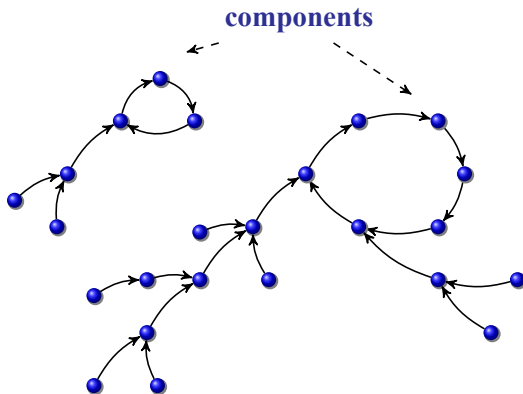
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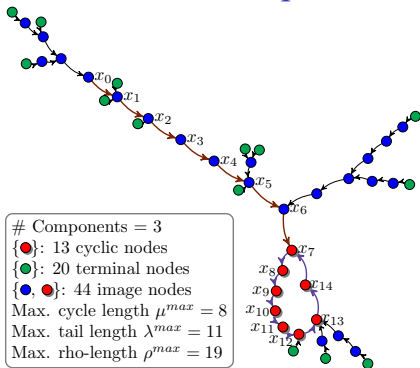




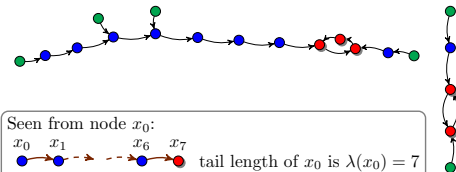
Preliminaries



Statistical Properties of Functional Graph [FO89]



Components = 3
 {●}: 13 cyclic nodes
 {●}: 20 terminal nodes
 {●, ●}: 44 image nodes
 Max. cycle length $\mu^{max} = 8$
 Max. tail length $\lambda^{max} = 11$
 Max. rho-length $\rho^{max} = 19$



Seen from node x_0 :

$x_0 \rightarrow x_1 \rightarrow \dots \rightarrow x_6 \rightarrow x_7$ tail length of x_0 is $\lambda(x_0) = 7$

$x_7 \rightarrow x_{14} \rightarrow x_{11} \rightarrow x_8 \rightarrow x_7$ cycle length of x_0 is $\mu(x_0) = 8$

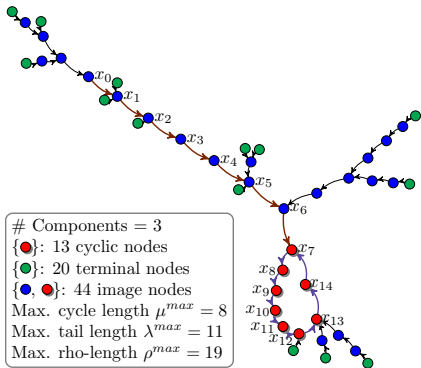
rho-length of x_0 is $\rho(x_0) = \lambda(x_0) + \mu(x_0) = 15$

- # Components: $0.5 \cdot n$
- # Cyclic nodes: $1.2 \cdot 2^n / 2$
- # Terminal nodes: $0.37 \cdot 2^n$

- # Image nodes: $0.62 \cdot 2^n$
- # k -th iterate image nodes: $(1 - \tau_k)N$
 where the τ_k satisfies the recurrence
 $\tau_0 = 0, \tau_{k+1} = e^{-1 + \tau_k}$.



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- Tail length (λ): $0.62 \cdot 2^{n/2}$
- Cycle length (μ): $0.62 \cdot 2^{n/2}$
- Rho-length (ρ): $1.2 \cdot 2^{n/2}$

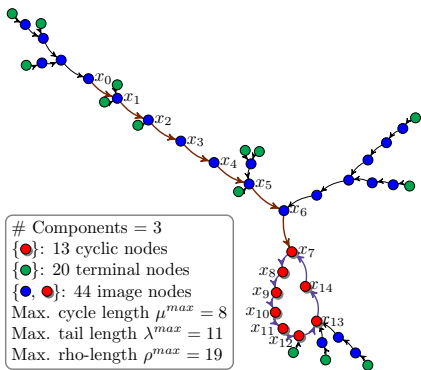
- Tree size: $0.34 \cdot 2^n$
- Component size: $0.67 \cdot 2^n$
- Predecessors size: $0.62 \cdot 2^{n/2}$



Preliminaries



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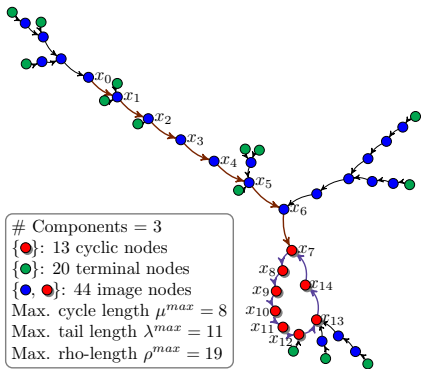
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- r -nodes: $N \cdot e^{-1}/r!$
- r -predecessor trees: $N \cdot t_r e^{-1}/r!$
- r -cycle trees: $\sqrt{\pi N/2} \cdot t_r e^{-1}/r!$

- r -cycles: $1/r$
- r -components: $c_r e^{-r}/r!$



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rho-length of x_0 is $\rho(x_0) = \lambda(x_0) + \mu(x_0) = 15$

- $\mathbf{E}\{\mu^{max} \mid \mathcal{F}_N\} = 0.78 \cdot 2^{n/2}$
- $\mathbf{E}\{\lambda^{max} \mid \mathcal{F}_N\} = 1.74 \cdot 2^{n/2}$
- $\mathbf{E}\{\rho^{max} \mid \mathcal{F}_N\} = 2.41 \cdot 2^{n/2}$
- $\mathbf{E}\{\text{tree}^{largest} \mid \mathcal{F}_N\} = 0.48 \cdot 2^n$
- $\mathbf{E}\{\text{component}^{largest} \mid \mathcal{F}_N\} = 0.76 \cdot 2^n$

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Attacks on Hash-based MAC Based on FG

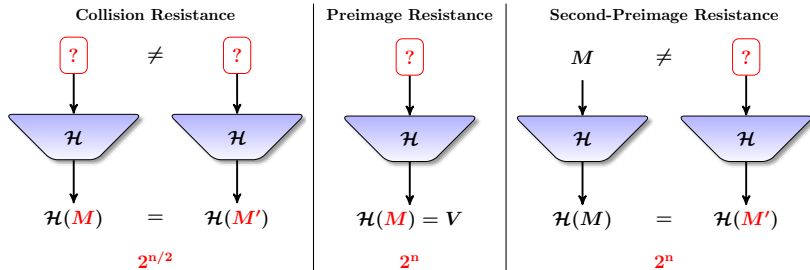
Attacks on Hash Combiners Based on FG

Summary and Open Problems



Cryptographic Hash Functions

- A hash function $\mathcal{H} : \{0, 1\}^* \rightarrow \{0, 1\}^n$ maps a message of arbitrary length to a digest of fixed length n -bit.

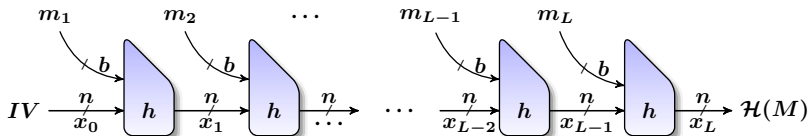


Credit: Bart Preneel



Underlying Construction - Iterative Hash Functions

- The Merkle-Damgård construction (MD) [Mer89; Dam89]:
Padding and dividing $M = m_1 || m_2 || \dots || m_L$, m_L is encoded with $|M|$ (length padding or Merkle-Damgård strengthening):



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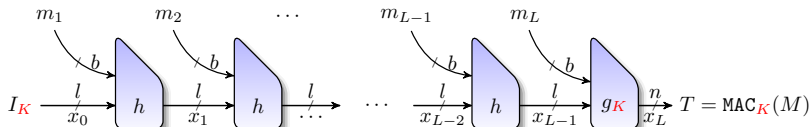
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Summary and Open Problems



Hash-based MACs

- Message Authentication Codes (MACs): symmetric method to provide authenticity
- One approach: Use hash functions with key K



Credit: [LPW13]



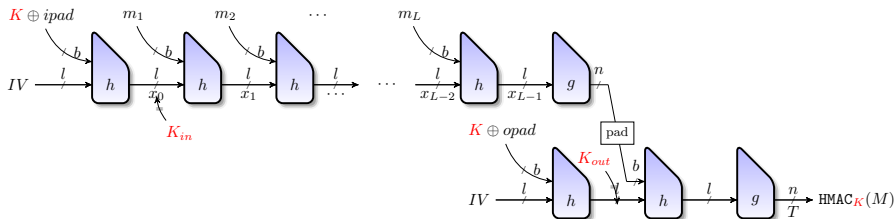
Hash-based MACs - Two Classical Designs

- NMAC:

$$\text{NMAC}(K_{out}, K_{in}, M) = \mathcal{H}_{K_{out}}(\mathcal{H}_{K_{in}}(M)).$$

- HMAC:

$$\text{HMAC}(K, M) = \mathcal{H}(K \oplus opad \parallel \mathcal{H}(K \oplus ipad \parallel M)).$$



HMAC with a Merkle-Damgård hash function

Credit: [Guo+14]



Security Requirement for Hash-based MACs

- Key recovery resistance: recover the key $\geq 2^k$
- State recovery resistance: recover the state $\geq \min(2^k, 2^l)$
- Forgery resistance: forge a valid tag of $M \geq \min(2^k, 2^n)$
 - Existential forgery: M is chosen by the adversary
 - Selective forgery: M is committed on by the adversary
 - Universal forgery: M is given to the adversary as a challenge



Security Requirement for Hash-based MACs

- Distinguishing-R:
e.g. distinguish HMAC from a PRF
- Distinguishing-H:
e.g. distinguish HMAC-SHA1 from HMAC-PRF



Distinguishing-H (recall)

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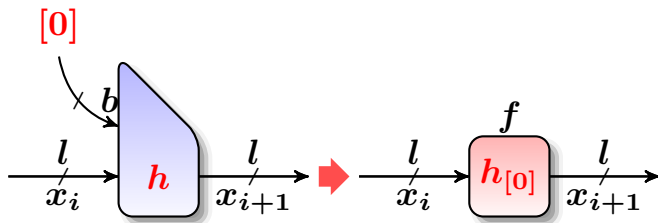
$$Adv(\mathcal{A}) = \left| \Pr \left[\mathcal{A}(\text{MAC}_K^h) = 1 \right] - \Pr \left[\mathcal{A}(\text{MAC}_K^r) = 1 \right] \right|.$$



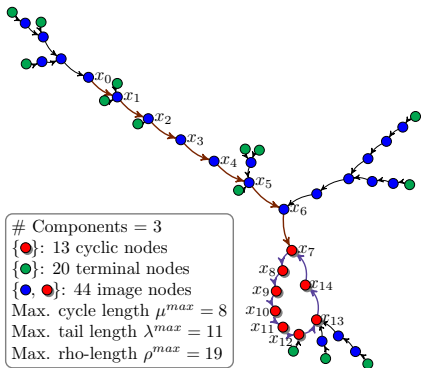
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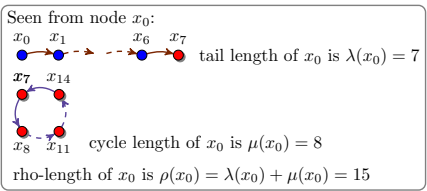
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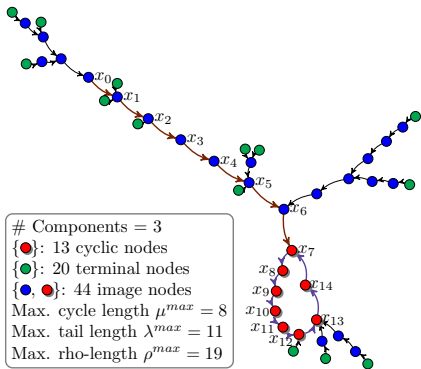


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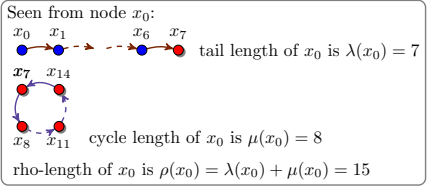


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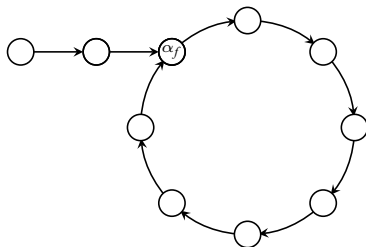


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Cycle-based Distinguishing-H Attack [LPW13]

○ → ○ offline of $h_{[0]} \mu$

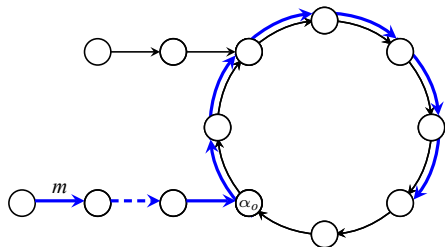




Cycle-based Distinguishing-H Attack [LPW13]

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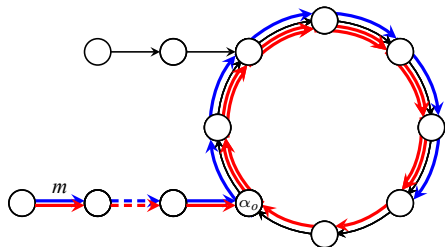


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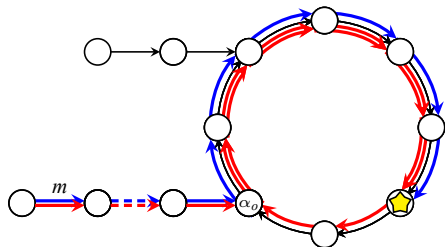
○ → ○ online $M_2 = m \parallel [0]^{2^{l/2} + \mu}$





Cycle-based Distinguishing-H Attack [LPW13]

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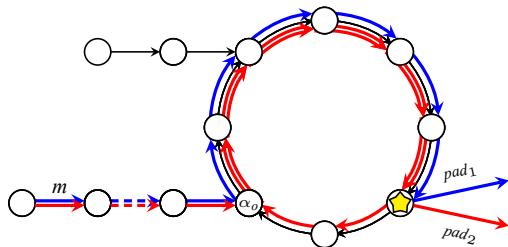


$$0.76 \times 1/2$$



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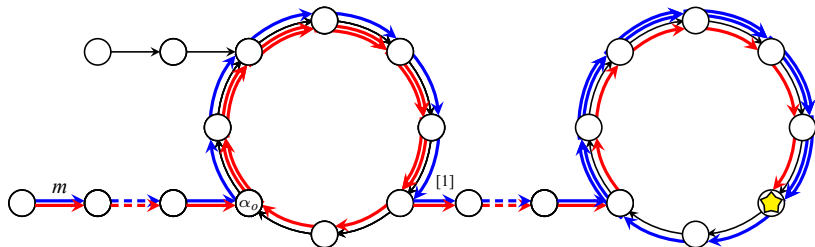


Cycle-based Distinguishing-H Attack [LPW13]

\rightarrow offline of $h_{[0]} \mu$

$\xrightarrow{\text{blue}}$ online $M_1 = m \parallel [0]^{2^{l/2}} \parallel [1] \parallel [0]^{2^{l/2} + \mu}$

$\xrightarrow{\text{red}}$ online $M_2 = m \parallel [0]^{2^{l/2} + \mu} \parallel [1] \parallel [0]^{2^{l/2}}$



$0.76 \times 1/2$

$\times 0.76 \times 1/2 \approx 0.14$

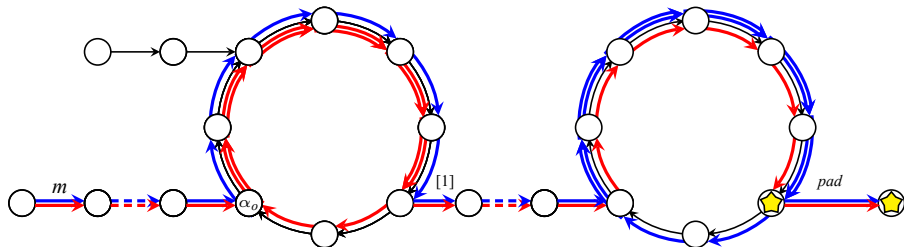


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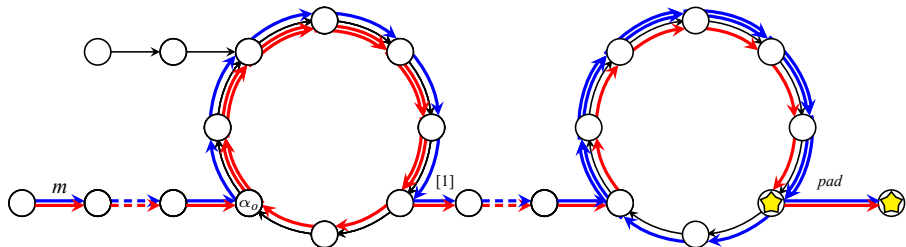


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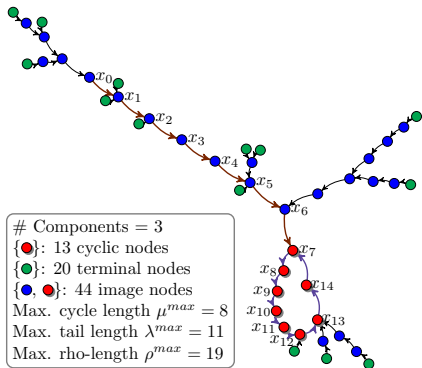


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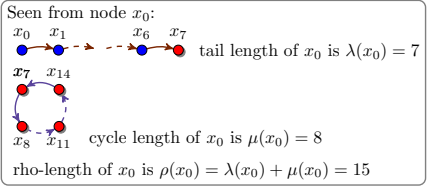
$$\times 0.76 \times 1/2 \approx 0.14$$

$$Adv(\mathcal{A}) = |0.14 - 2^{-l/2}| \approx 0.14$$

Statistical Properties of Functional Graph [FO89] (recall)



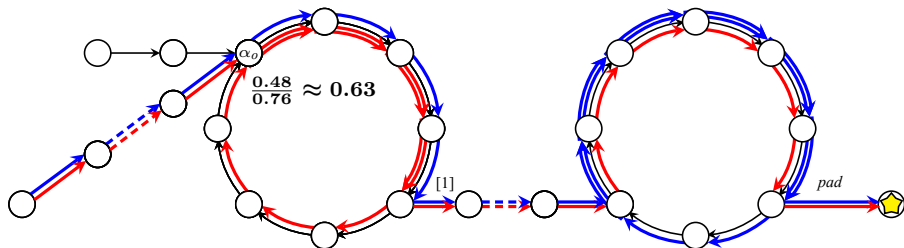
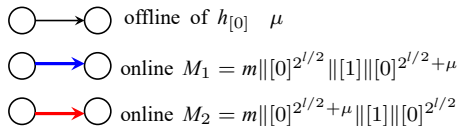
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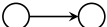




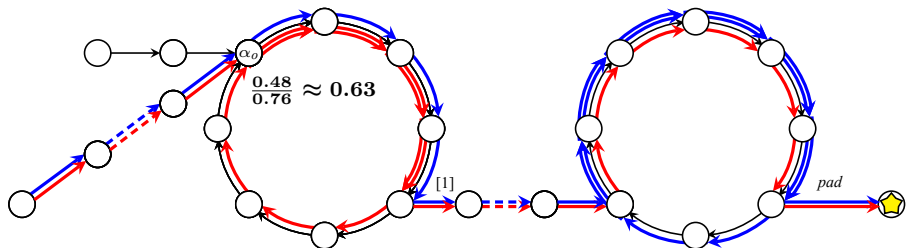
Cycle-based State Recovery Attack [LPW13]





Cycle-based State Recovery Attack [LPW13]

-  offline of $h_{[0]}$ μ Binary Search: 1. $X_1 \leftarrow 0, X_2 \leftarrow 2^{l/2}$
-  online $M_1 = m \parallel [0]^{X'} \parallel [i] \parallel [0]^{2^{l/2} + \mu}$ 2. $X' \leftarrow (X_1 + X_2)/2$, query with M_1 and M_2
-  online $M_2 = m \parallel [0]^{X'} + \mu \parallel [i] \parallel [0]^{2^{l/2}}$ 3. $X_2 \leftarrow X'$ if collide, $X_1 \leftarrow X'$ other, go to 2.

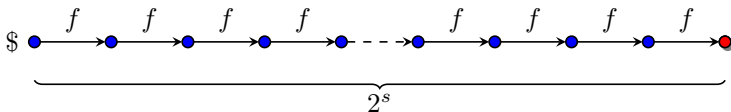


Entropy Loss of Chain Evaluation

Lemma 1 ([DL17], Lemma 1)

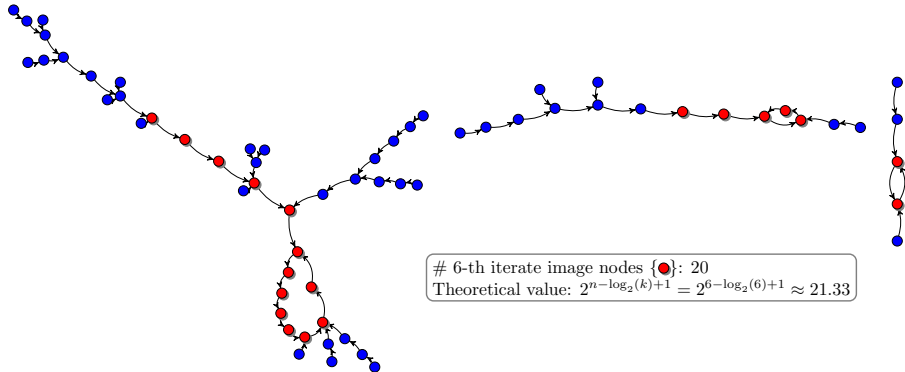
Let $s \leq l/2$ be a non-negative integer. Let f be a random function over the set of 2^l elements. Then, the images of two arbitrary inputs to f^{2^s} collide with probability of about 2^{s-l} , i.e.,

$$\Pr_{x,y}[f^{2^s}(x) = f^{2^s}(y)] = \Theta(2^{s-l}).$$





Statistical Properties of Functional Graph [FO89] (recall)



A k -th iterate image node in the functional graph of a random mapping $f \in \mathcal{F}_N$ is an image of the k -th iterate f^k of f .

k -th iterate image nodes $(1 - \tau_k)N$, where the τ_k satisfies the recurrence $\tau_0 = 0, \tau_{k+1} = e^{-1+\tau_k}$.



The Expected Number of k -th Iterate Image Nodes in FG

Lemma 2

Let f be a random mapping in \mathcal{F}_N . Denote $N = 2^n$. For $k \leq 2^{n/2}$, the expectation of number of k -th iterate image nodes in the functional graph of f is

$$(1 - \tau_k) \cdot N \approx \left(\frac{2}{k} - \frac{2 \log k}{3 k^2} - \frac{c}{k^2} - \dots \right) \cdot N.$$

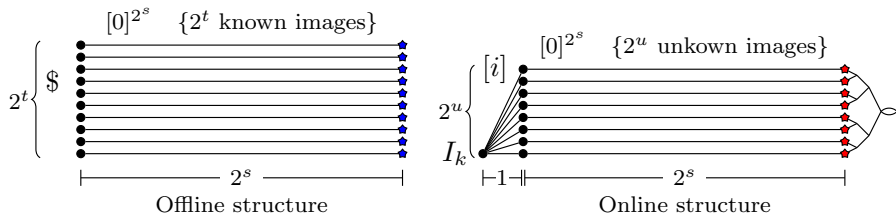
It suggests that $\lim_{k \rightarrow \infty} k \cdot (1 - \tau_k) = 2$. Thus,

$$\lim_{N \rightarrow \infty, k \rightarrow \infty, k \leq \sqrt{N}} (1 - \tau_k) \cdot N \approx 2^{n - \log_2(k) + 1},$$

where τ_k satisfies the recurrence $\tau_0 = 0$, $\tau_{k+1} = e^{-1 + \tau_k}$, and c is a certain constant.



State Recovery Attack Based on Reduction of Image-set Size [DL17]



$$t + u = l - s$$

We detect (off-line) a match between 2^t off-line known states (\blacklozenge) with 2^u on-line unknown states (\blackstar) using the diamond filter built on-line.

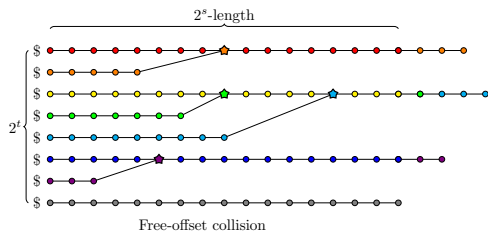
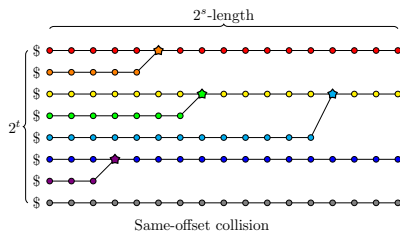
Step 1: $2^{t+s} = 2^{l-u}$ **Step 2:** $2^{u+s} + u \cdot 2^{s+u/2+l/2}$

Step 3: $2^{t+u} \cdot u = 2^{l-s} \cdot u$ Total complexity: $\tilde{O}(2^{l-s})$ for $s \leq l/5$;

Optimal complexity $4l/5$ when $s = l/5$.



Entropy Loss of Collision Search [LPW13; DL17]

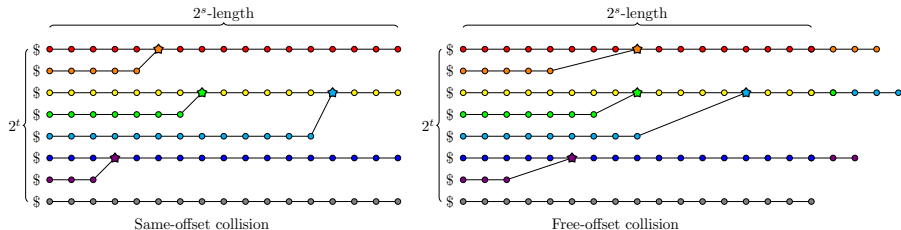


Suppose the iteration functions are all identical, and $2^{t+2s} \leq 2^l$

- For same-offset collisions:
- Expected number: 2^{2t+s-l}
- Complexity to get 2^c : $2^{l/2+s/2+c/2}$

- For free-offset collisions:
- Expected number: $2^{2(t+s)-l}$
- Complexity to get 2^c : $2^{l/2+c/2}$

Entropy Loss of Collision Search [LPW13; DL17]

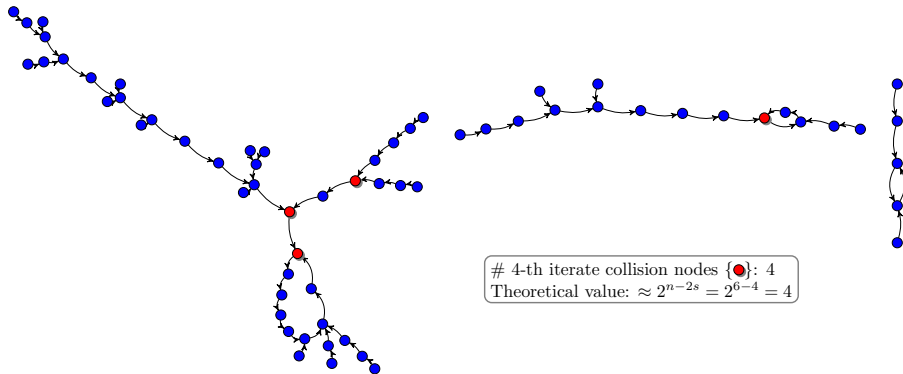


Lemma 3 ([DL17], Lemma 3)

Let \hat{x} and \hat{y} be two random collisions found by a collision search algorithm using 2^t chains of length 2^s , with a fixed l -bit random function f such that $2s + t \leq l$. Then $\Pr[\hat{x} = \hat{y}] = \Theta(2^{2s-l})$.



The Expected Number of k -th Iterate Collision Nodes



Definition 4 (k -th iterate collision node)

A k -th iterate collision node in the functional graph of a random mapping $f \in \mathcal{F}_N$, is an r -node (a node of in-degree r), where $r \geq 2$ and at least two of its pre-images are k -th iterate image nodes.



The Expected Number of k -th Iterate Collision Nodes

Theorem 5 ([FO89])

The expected number of r -nodes (a node of in-degree r) is $N \cdot e^{-1}/r!$.

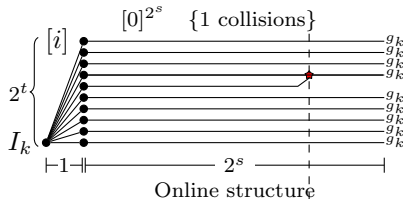
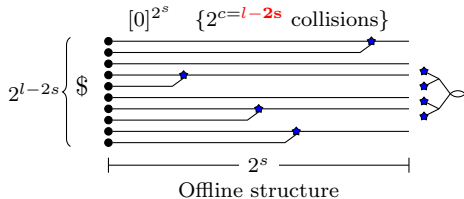
The expected total number of collision nodes (0-th iterate collision nodes) in the functional graph of a random mapping $f \in \mathcal{F}_N$ is $(1 - 2 \cdot e^{-1}) \cdot N = 0.2642 \cdot N$.

Lemma 6

Denote $N = 2^n$. For $N \rightarrow \infty$, $k \rightarrow \infty$ and $k \leq 2^{n/2}$, the expected number of k -th iterate collision nodes in the functional graph of a random mapping $f \in \mathcal{F}_N$ is $\Theta(k^{-2} \cdot N)$.



State Recovery Attack Based on Collisions [DL17]



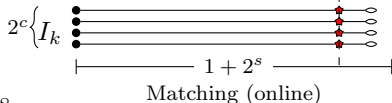
Step 1: $2^{l-s} + 2^{(c+l)/2} \approx 2^{l-s}$

Step 2: $2^{t+s} + s \cdot 2^s = 2^{(l+s)/2} + s \cdot 2^s$

Step 3: $2^{c+s} = 2^{l-s}$

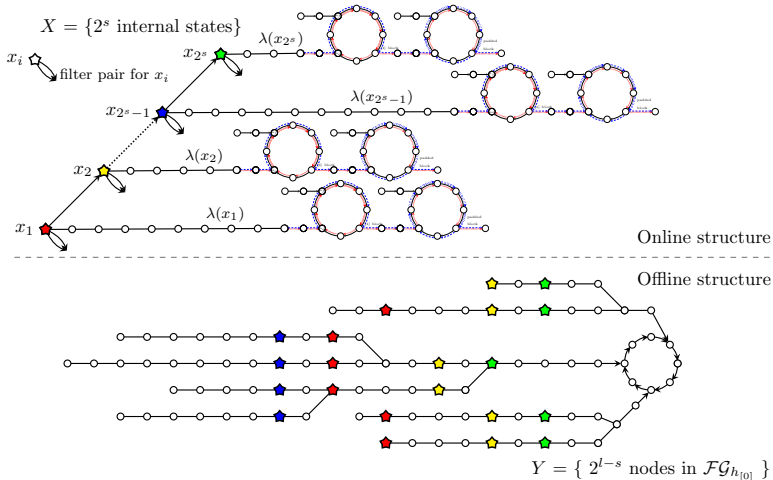
Total complexity: $O(2^{l-s})$ if $s \leq l/3$

Optimal complexity: $\tilde{O}(3l/4)$ when $s = l/8$.





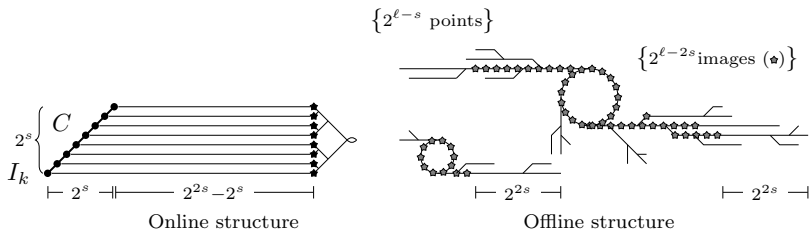
Universal Forgery Attacks Based on Cycles and Height [PW14; Guo+14]



Only match elements in X and elements in Y at same height (same color implying same height).



Universal Forgery Attacks Based on Chain and Collisions [DL14; DL17]



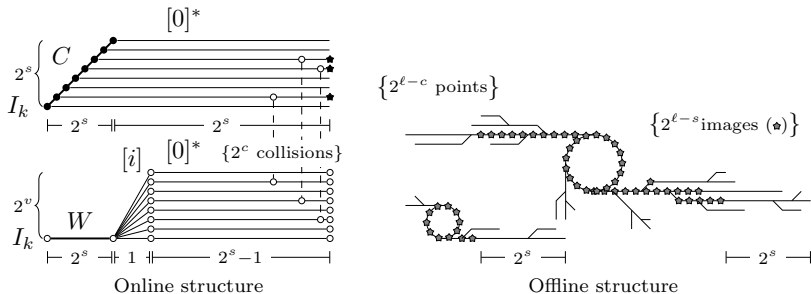
We efficiently detect a match between the challenge points (\bullet) and the offline structure, by first matching X (\star) and Y (\star).

Total complexity: $\tilde{O}(2^{l-s})$ for any $s \leq l/7$.

Optimal complexity: $2^{6l/7}$, obtained when $s = l/7$.



Universal Forgery Attacks Based on Chain and Collisions [DL14; DL17]



We match the known points in X (\blacklozenge) and Y (\blacklozenge) in order to detect a match between the challenge points (\bullet) and the offline structure.

Total complexity: $\tilde{O}(2^{\ell-s/2})$ for any $s \leq 2l/5$.

Optimal complexity: $2^{4l/5}$, when $s = 2l/5$.

Outline

Functional Graph

Preliminaries

Attacks on Hash-based MAC Based on FG

Attacks on Hash Combiners Based on FG

Summary and Open Problems



Hash Combiners

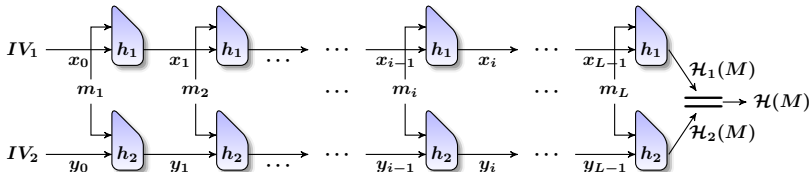
An approach to construct a secure hash function

- Security amplification
the combiner is more secure than its underlying hash functions;
- Security robustness
the combiner is secure as long as any one of its underlying hash functions is secure

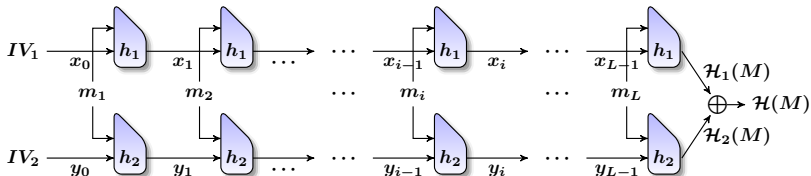


Hash Combiners - Parallel

- Concatenation combiner:



- XOR combiner:





Expected Security of Hash Combiners Before 2004

	Digest Size	Collision Resistance	Preimage Resistance	Second Preimage Resistance
Ideal \mathcal{H}	n	$2^{n/2}$	2^n	2^n
Ideal $\mathcal{H}_1 \parallel \mathcal{H}_2$	$2n$	2^n	2^{2n}	2^{2n}
Ideal $\mathcal{H}_1 \oplus \mathcal{H}_2$	n	$2^{n/2}$	2^n	2^n

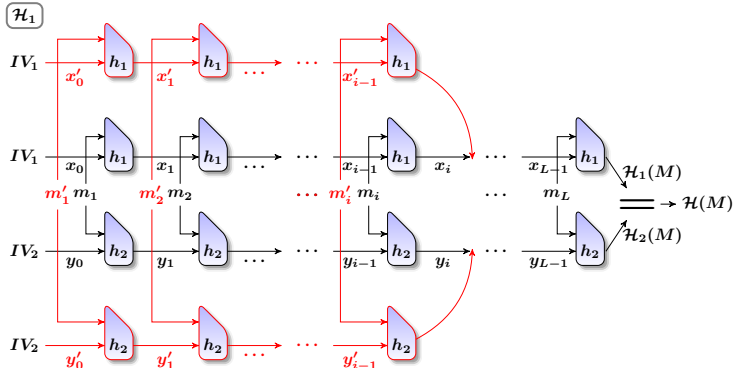
↑
birthday bound
half of digest size

↑
full digest size

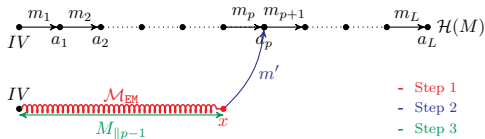


Second-Preimage Attack on Concatenation Combiner

Goal:

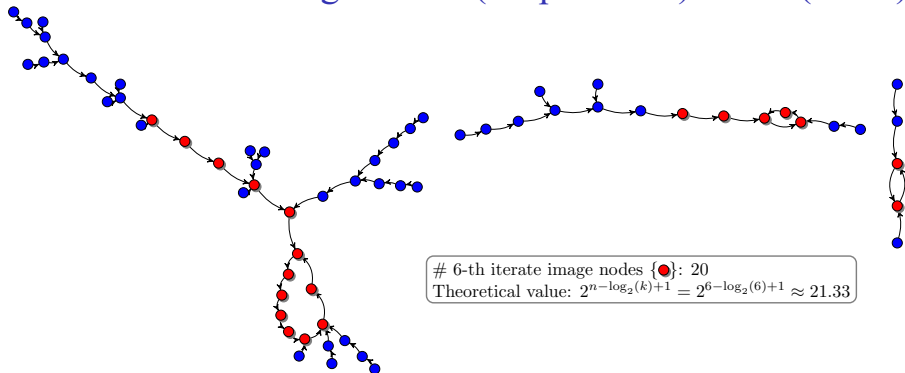


\mathcal{H}_2





The k -th Iterate Image Nodes (deep iterates) in FG (recall)

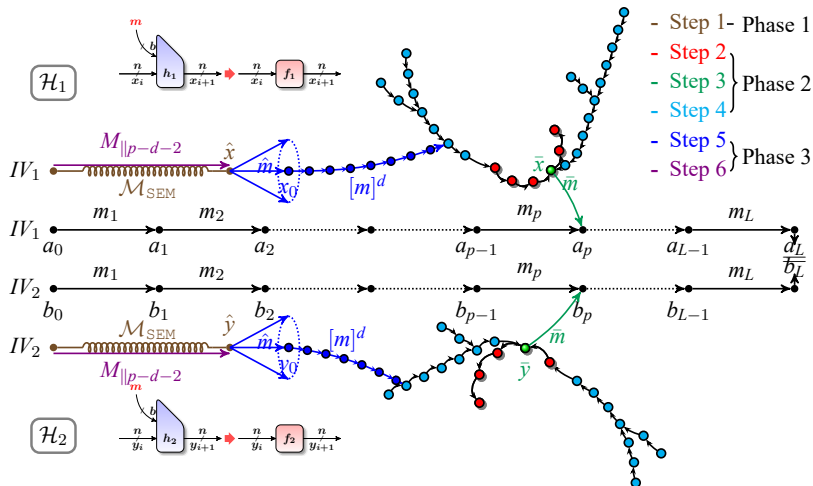


The expectation of number of k -th iterate image nodes is $\approx 2^{n-\log_2(k)+1}$

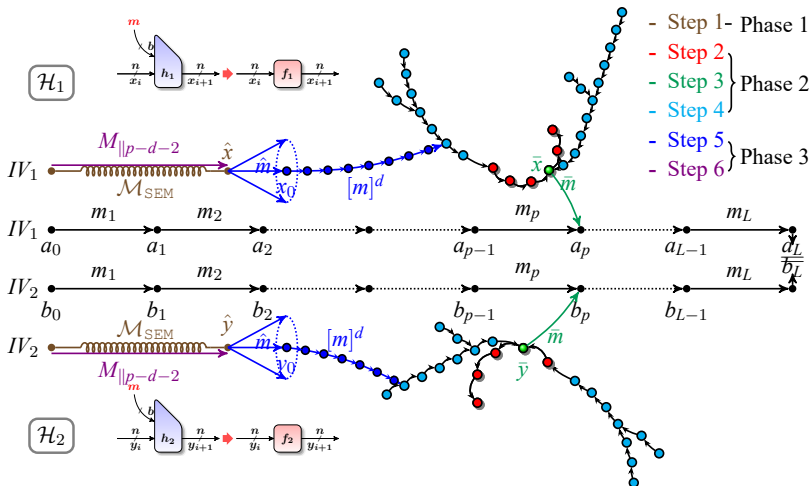
Lemma 7

Let f be an n -bit random mapping, and x'_0 an arbitrary point. Let $D \leq 2^{n/2}$ and define the chain $x'_i = f(x'_{i-1})$ for $i \in \{1, \dots, D\}$. Let x_0 be a randomly chosen point, and define $x_d = f(x_{d-1})$. Then, for any $d \in \{1, \dots, D\}$, $\Pr[x_d = x'_d] = \Theta(d \cdot 2^{-n})$.

Second-Preimage Attack Based on Deep Iterates [Din16]

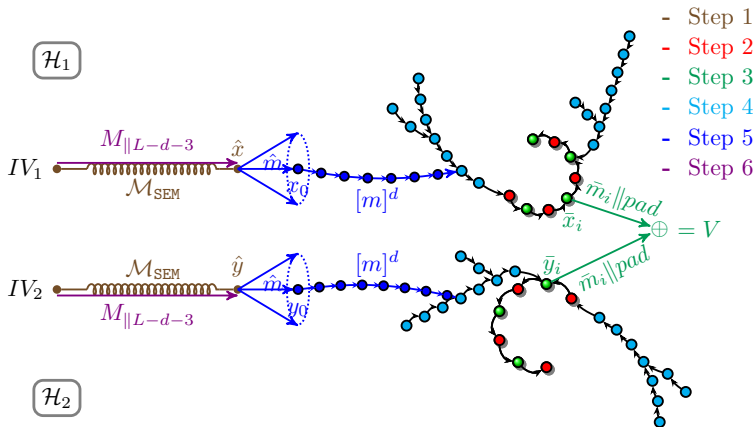


Second-Preimage Attack Based on Deep Iterates [Din16]



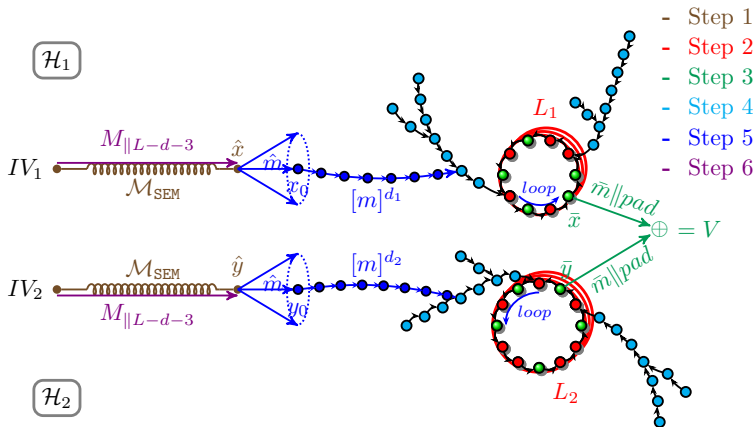
Phase 1: $2^l + n^2 \cdot 2^{n/2}$ **Phase 2:** 2^{n+g-l} **Phase 3:** $2^{3n/2-3g/2}$
 (use 2^g -deep iterates, set $g = n/5 + 2l/5$. Total: $2^{6n/5-3l/5}$ if $l < 3n/4$)

Preimage Attack on XOR Combiner Based on Deep Iterates [Din16]



Optimal complexity: $2^{2n/3}$, obtained when $l = n/2$.

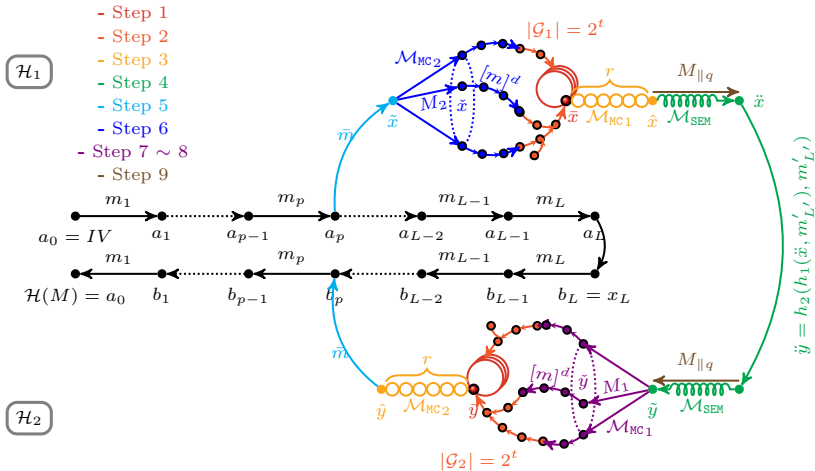
Preimage Attack on XOR Combiner Based on Multi-Cycles [Bao+17]



Optimal complexity: $2^{5n/8}$, obtained when $l = 5n/8$.



Second-Preimage Attack on Zipper Hash Based on Multi-Cycles [Bao+17]



Optimal complexity: $2^{3n/5}$, obtained when $l \geq 2n/5$ and $l' = 3n/5$.

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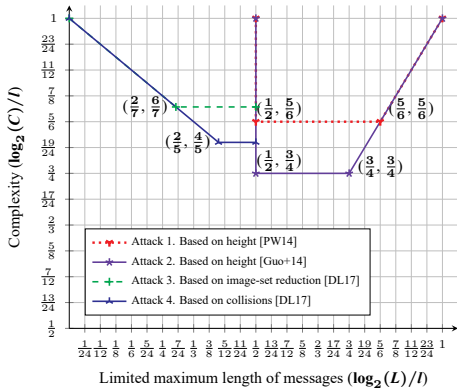
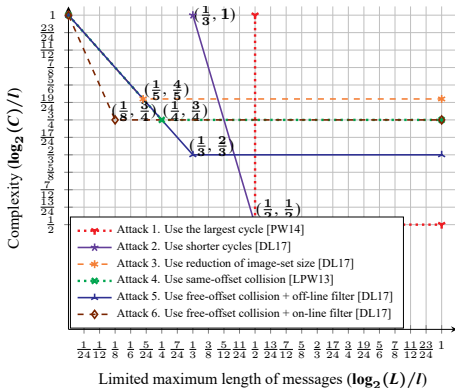


Relations Between Properties Utilized in Various Attacks and Properties of Functional Graphs

- Cycle search algorithm
 - output the cycle length and cyclic nodes
 - two outputs collide with constant probability
 - entropy loss is about l bits
- Chain evaluation algorithm
 - output deep (2^s) iterate nodes
 - two outputs collide with probability 2^{s-l}
 - entropy loss is about s bits
- Collision search algorithm
 - output deep (2^s) collision nodes
 - two outputs collide with probability 2^{2s-l}
 - entropy loss is about $2s$ bits

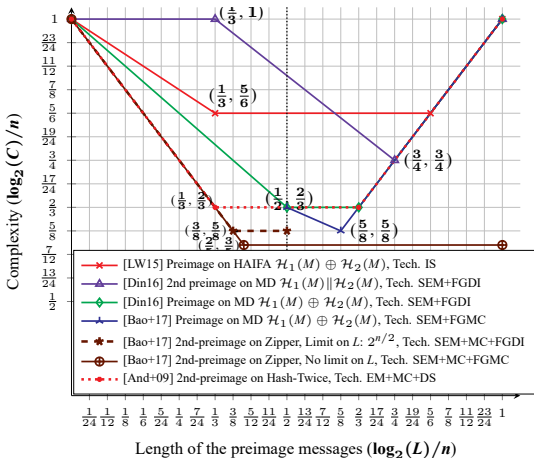


Summary on Generic Attacks against Hash-based MACs





Summary on Generic Attacks against Hash Combiners





Remarks on Approaches from Analytic Combinatorics

- Approaches from analytic combinatorics – the symbolic method, generating functions, and asymptotic analysis
- Is it possible to use analytic combinatorics to directly get asymptotic formulas for more special parameters (e.g., the expected number of k -th iterate collision nodes)?
- Is it possible to build combinatorial models for other concerned objects in cryptanalysis (e.g., the partial functional graph restored by some probabilistic algorithm)?



Thanks for your attention!

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