Boomerang Switch in Multiple Rounds

Application to AES Variants and Deoxys

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FSE 2019, Paris

March 26, 2019

Outline

- Background
- Boomerang Switch
- Attack on 10-round AES-256
- Application to Full-round AES-192 and reduced-round Deoxys-BC

Background

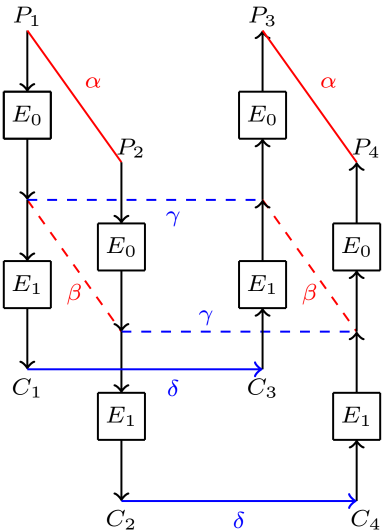
Boomerang Attack

Boomerang attack

- A cipher E is divided into two sub-ciphers:

$$E = E_1 \circ E_0$$

- E_0: $P[\alpha \rightarrow \beta] = p$
- E_1: $P[\gamma \rightarrow \delta] = q$
- The two trails are assumed to be independent.
- Distinguish probability: $Pr[E^{-1}(E(x) \oplus \delta) \oplus E^{-1}(E(x \oplus \alpha) \oplus \delta) = \alpha] = p^2q^2$



Dependency Between the Two Sub-Ciphers

- At the boundary of the two trails, dependency may exist.

Positive effect

- Middle round S-box trick [BDD03]
- Ladder switch [BK09]
- S-box switch [BK09]
- Feistel switch [BK09]

Negative effect

- Imcompatibility [Mer09]

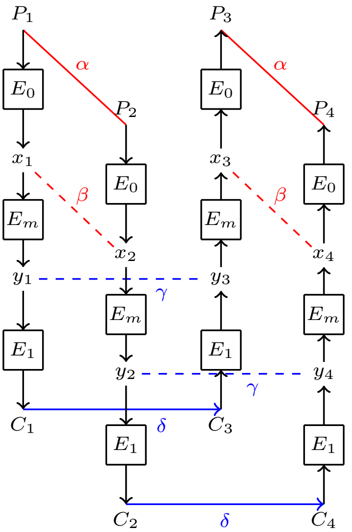
Sandwich Attack

Sandwich attack

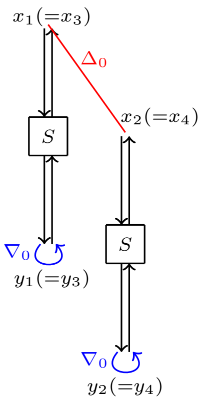
- E is further divided into three sub-ciphers:

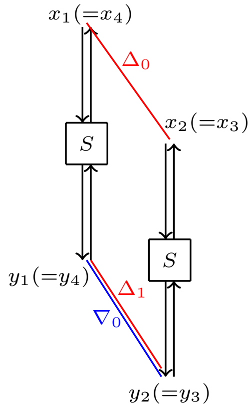
$$E = E_1 \circ E_m \circ E_0$$

- E_m contains the dependent parts of the two trails, with probability r
- $r = Pr[E_m^{-1}(E_m(x) \oplus \gamma) \oplus E_m^{-1}(E_m(x \oplus \beta) \oplus \gamma) = \beta]$
- Distinguish probability: p^2q^2r.



View of Boomerang Switch in Sandwich Attack





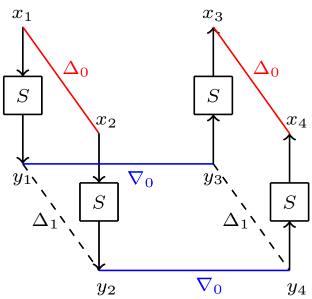
Ladder switch

1. $\nabla_0 = 0$
2. $y_3 = y_1$ and $y_4 = y_2$
3. $x_3 = x_1$ and $x_4 = x_2$
4. $r = 1$

Sbox switch

1. $\nabla_0 = \Delta_1$
2. $y_4 = y_1,\ y_3 = y_2$
3. $x_4 = x_1$ and $x_3 = x_2$
4. $r = pr[\Delta_0 \xrightarrow{Sbox} \Delta_1]$

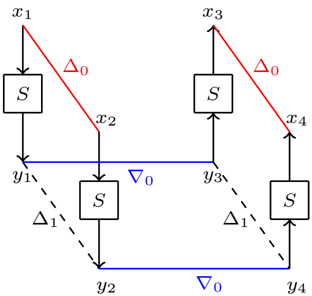
Boomerang Connectivity Table (BCT)



Construction

- Focus on a single S-box layer.
- Δ_0 and ∇_0 are taken into consideration.
- The entry for (Δ_0, ∇_0) is computed by $\#\{x \in \{0,1\}^n | S^{-1}(S(x) \oplus \nabla_0) \oplus S^{-1}(S(x \oplus \Delta_0) \oplus \nabla_0)\}$.

Boomerang Connectivity Table (BCT)



Advantages

- It covers the switching effect of ladder switch, S-box switch and incompatibility.
- New switching effect: Compared to S-box switch where $\nabla_0 = \Delta_1$, BCT does not require the value of Δ_1, which could lead to a higher switching probability.

Motivation

Questions

- Can we extend E_m to multiple rounds?
- If yes, can current switching techniques be applied to the multiple-round case?

Boomerang Switch

Determining the Number of Rounds in E_m

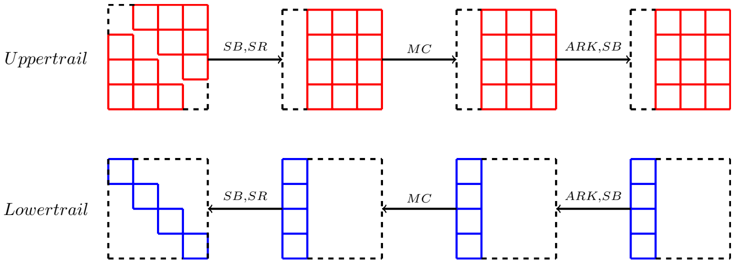


Figure: Parallel operations of truncated 2-round AES

The idea of ladder switch

The round function of a cipher can be divided into two independent parts, which can operate in parallel.

Extension

In E_m, if the forward diffusion of the active cells in the upper trail has no interaction with the backward diffusion of the active cells in the lower trail, a right quartet of E_m can be generated with probability 1.

Determining the Number of Rounds in E_m

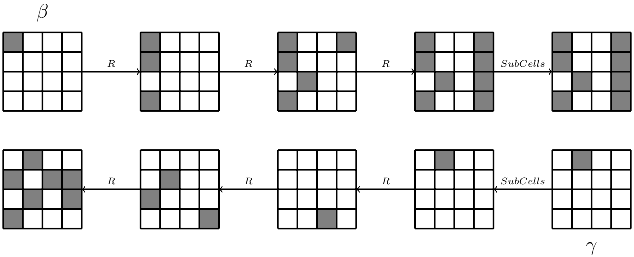


Figure: A 4-round E_m of SKINNY with probability 1

Observation

- For SKINNY [BJK+16], E_m can be at most four rounds with probability $r = 1$.
- E_m contains more rounds for those ciphers with slower diffusion layer.

Incompatibility in Multiple Rounds

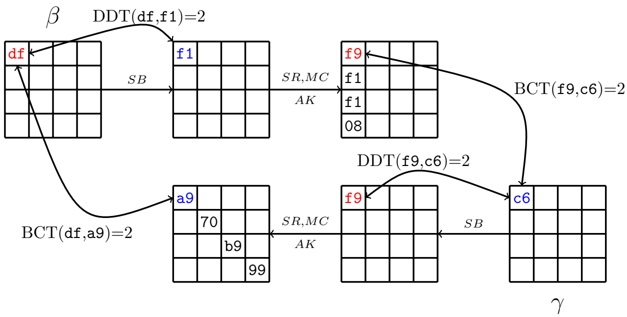
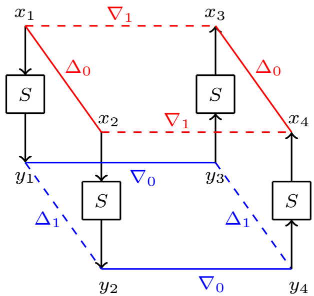


Figure: An incompatible 2-round E_m of AES

Deficiency of BCT

- BCT detects incompatibility while the entry is zero.
- The two trails are valid with probability 2^{-7} respectively: DDT(df,f1)=2, DDT(f9,c6)=2.
- For the two active S-boxes, the entries of BCT are non-zero: BCT(df,a9)=2, BCT(f9,c6)=2.
- However, this example is incompatible: BCT(df,a9) and DDT(df,f1) cannot be non-zero simultaneously.

Observation on S-box in the Boomerang Switch



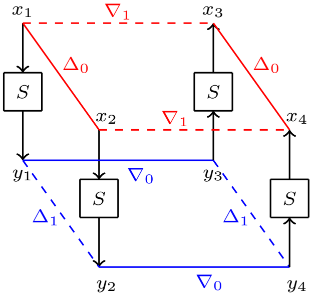
Lemma1

For any fixed Δ_0 and Δ_1, for which the DDT entry is $2l$, l being a nonzero integer, the maximum number of nontrivial values of ∇_0, for which a right quartet could be generated, is $2\binom{l}{2} + 1$.

Lemma2

For any fixed Δ_0 and ∇_0, for which the BCT entry is $2l$ and the DDT entry is $2l'$, l and l' being nonzero integers, the maximum number of choices of Δ_1, for which a right quartet could be generated, is $1 + (2l - 2l')/4$.

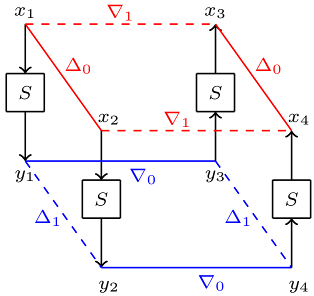
Boomerang Difference Table (BDT)



Construction

- A combination of BCT and DDT.
- The entry for $(\Delta_0, \Delta_1, \nabla_0)$ is defined by: $\#\{x \in \{0,1\}^n | S^{-1}(S(x) \oplus \nabla_0) \oplus S^{-1}(S(x \oplus \Delta_0) \oplus \nabla_0) = \Delta_0, S(x) \oplus S(x \oplus \Delta_0) = \Delta_1\}$, n is the S-box size.
- The time complexity for the construction is $O(2^{2n})$.

Boomerang Difference Table (BDT)



Properties

- $DDT(\Delta_0, \Delta_1) = BDT(\Delta_0, \Delta_1, 0) = BDT(\Delta_0, \Delta_1, \Delta_1)$
- $BCT(\Delta_0, \nabla_0) = \sum_{\Delta_1=0}^{2^n} BDT(\Delta_0, \Delta_1, \nabla_0)$
- $BDT(0, 0, \nabla_0) = 2^n$
- $(\Delta_0, \Delta_1, \nabla_0)$ is incompatible when the corresponding entry in BDT is 0.

Attack on 10-round AES-256

Attack model

Related-key attack

- The adversary chooses a relation between several keys, e.g., $K_2 = K_1 \oplus C$ and is given access to encryption/decryption oracles with these keys.

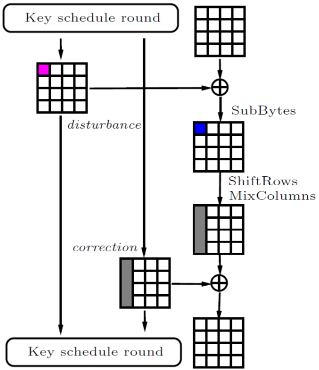
Related-subkey attack

- The adversary chooses a relation between subkeys, e.g., $K_2 = F^{-1}(F(K_1) \oplus C)$, where F represents the round function of key schedule.
- Advantage: easier to obtain a desired related-subkey difference in non-linear key schedule.
- Disadvantages: complex key access scheme, less practical and even too contrived for academic interest.

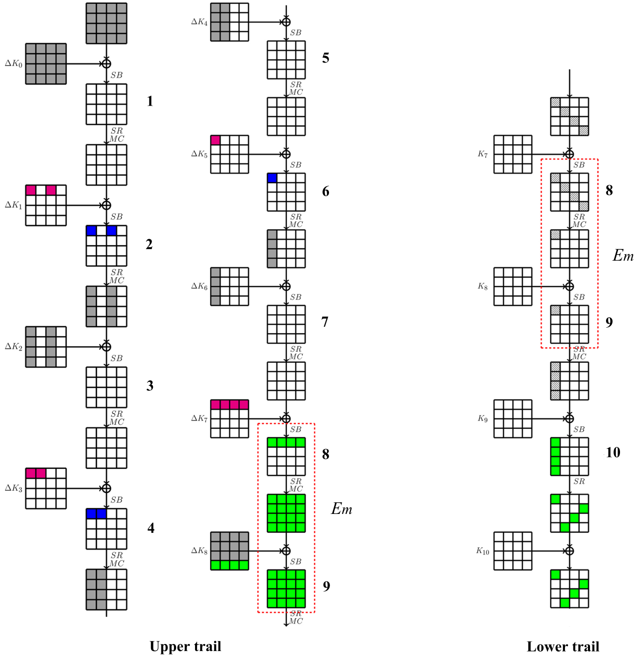
Overview of the Attack

Idea

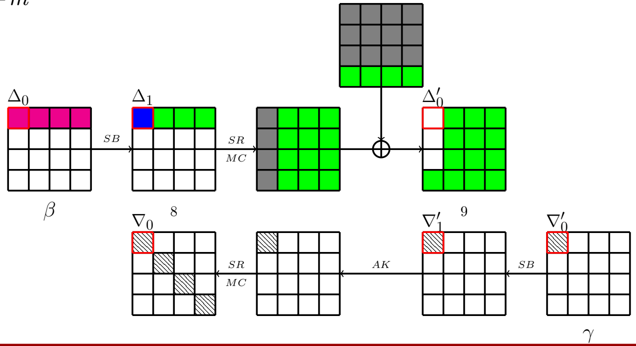
- We stick to the related-key attack. Since the key schedule of AES is non-linear, a related-key differential path is used for the upper trial while a single-key differential path is used for the lower trail.
- The local collision strategy is used for constructing the upper trail.
- Apply the boomerang switch in two rounds.



The 10-round Attack



The 2-round E_m



Analysis

- β and γ are fixed.
- For the S-box at (0,0) in round 8:
  - A fixed value Δ_1 is chosen so that there is no overlapped active cell in round 9.
  - With the fixed Δ_0 and Δ_1, choose the values of ∇_0 so that the BDT entries are non-zero, and the switching probability is obtained accordingly.
- For the S-box at (0,0) in round 9:
  - ∇'_1 is uniquely determined by ∇_0.
  - Since $\Delta'_0 = 0$, the switching probability can be evaluated by DDT with entry (∇'_1, ∇'_0)

Result

Scenario	# keys	Time	Data	Result	Reference
Key Diff.	64/256	2^{172}	2^{114}	Full key	[KHP07]/[BDK05]
Subkey Diff.	2	$2^{45}(2^{221})$	2^{44}	35 subkey bits (full key)	[BDK+10]
Key Diff.	2	2^{75}	2^{75}	Full key	this paper

Application to Full-round AES-192 and reduced-round Deoxys-BC

Overview of the Previous Attacks

- Full-round AES-192 [BN09]: the first related-key boomerang attack on full-round AES-192.
- Full-round AES-192 [BN10]: the upper trail is different than [BN09], and remains as the best attack.
- 10-round Deoxys-BC[CHP+17]: its distinguisher is built with the idea of 2-round boomerang switch.

Improvement of the Attack [BN10]

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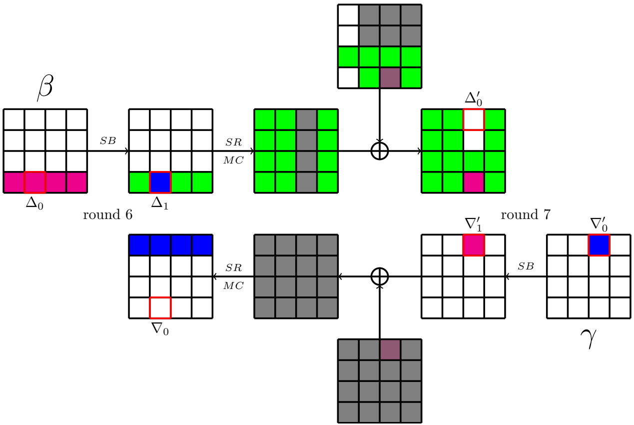
Idea

- The original attack [BN10] uses a similar idea of local collision. The boomerang switch is optimized in one round.
- With the help of BDT, we managed to extend the boomerang switch to 2-round by searching a new upper trail.

The 2-round E_m of the Improved Attack on [BN10]



Analysis

- No overlapped active S-box in the two S-box layer.
- However, specific values of Δ_1 and ∇'_1 are required.
- The switching probabilities of the corresponding two S-boxes are counted.

Results

Attacks	Improvement(Data&Time)
AES-192 [BN10]	$2^{1.3}$
AES-192 [BN09]	$2^{4.8}$
Deoxys-BC-256 [CHP+17]	$2^{1.6}$

Conclusion

- The slower is the diffusion in a cipher, the more rounds will be impacted by the switching effect.
- We introduced the BDT to easily evaluate the boomerang switch in multiple rounds.
- Improved attacks on 10-round AES-256, full-round AES-192 and reduced round Deoxys-BC-256.

THANK YOU!