Classical and Quantum Meet-in-the-Middle Nostradamus Attacks on AES-like Hashing

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Outline

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Background

Mr. Nostradamus and his friends passed by a lottery shop in Leuven several days ago. He said to his friends:
“I can predict the lottery numbers of March 27th, I have written them down in my diary. I won’t show you my diary now, but I could tell you the hash value of it.”

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Mr. Nostradamus sent a hash value $\mathcal{T}$ to his friends.

Background

After the winning numbers were announced,
Mr. Nostradamus showed his diary $\mathcal{D}$ to his friend.

The first line of the diary is:
"The lottery numbers of March 27th are 2 3 19 40 42 43 4."
Which is exactly the same as the winning numbers.
Furthermore, it could be verified that $H(\mathcal{D}) = \mathcal{T}$.

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How did Mr. Nostradamus do that?

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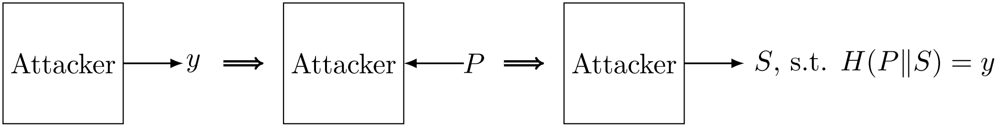
The Herding Attack [KK06]

The chosen target forced prefix (CTFP) attack was first introduced by Kelsey and Kohno at EUROCRYPT 2006. They proposed the herding attack on iterated hash functions.

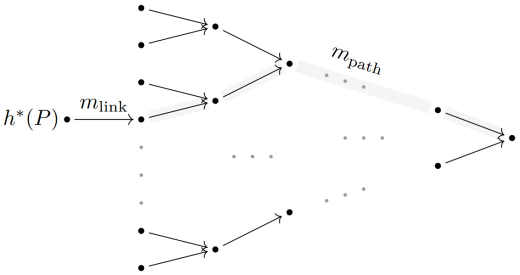


The "Nostradamus attack" is the use of herding to commit to the hash of a message that the attacker doesn't even know.

The Herding Attack [KK06]

The attack is divided into two phases:

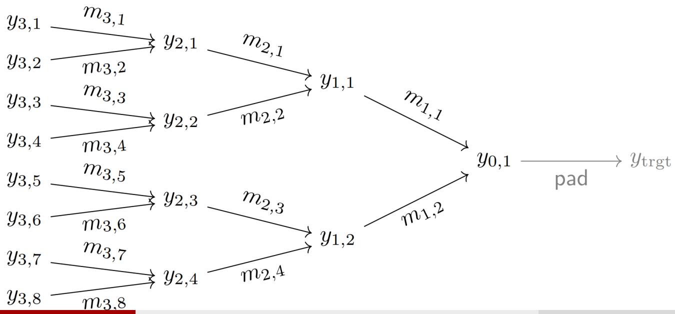
1. **Offline phase**: Construct a diamond structure.
2. **Online phase**: Search for the link message.



The Herding Attack [KK06]: Offline phase

Diamond structure: 2^k multi collisions

- Node $y_{i,j}$: the intermediate value of the Merkel-Damgård constructions.
- Edge $m_{i,j}$: the message block that links two intermediate value, $CF(y_{i,j}, m_{i,j}) = y_{i-1,\lceil j/2 \rceil}$
- Time complexity: $\mathcal{O}(2^{\frac{n+k}{2}})$. Memory complexity: $\mathcal{O}(2^k)$.



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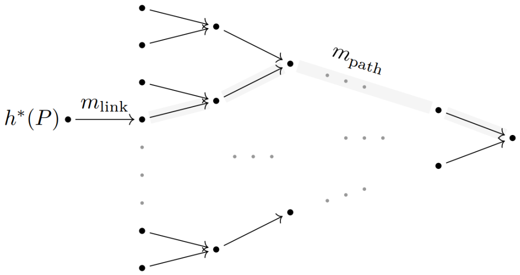
The Herding Attack [KK06]: Online phase

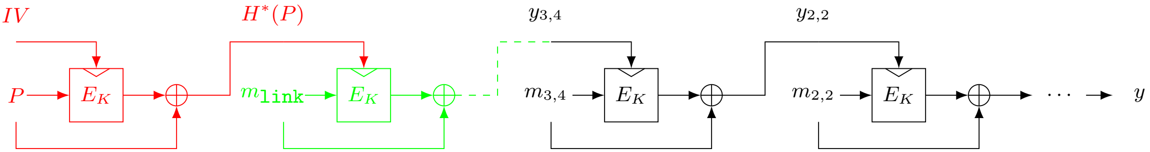
Once the prefix P is given, the attacker could exhaustive search for the link message, connecting the prefix with a leaf node.

Time complexity: $\mathcal{O}(2^{n-k})$. Memory complexity: $\mathcal{O}(2^k)$.



The Herding Attack [KK06]

To minimize the time complexity of both phases, the optimal choice of k is $\frac{n}{3}$.
Time complexity: $\mathcal{O}(\sqrt{n} \cdot 2^{2n/3})$. Memory complexity: $\mathcal{O}(2^{n/3})$.



Online Phase
Exhaustive Search

Offline Phase
Collision-based Method

The Quantum Nostradamus Attack

At ASIACRYPT 2022, Benedikt, Fischli, and Huppert [BFH22] followed the classical herding attack and use Grover-based quantum algorithms to accelerate both offline and online phases.

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Table: The Generic Nostradamus Attacks

Ref.	Time	c-Memory	qRam	Setting
[KK06]	$2^{\frac{2n}{3}}$	$2^{\frac{2n}{3}}$	0	Classical
[BFH22]	$2^{\frac{3n}{7}}$	0	$2^{\frac{n}{7}}$	Quantum
[Don+23]	$2^{\frac{6n}{13}}$	$2^{\frac{3n}{13}}$	$\mathcal{O}(n)$	Quantum

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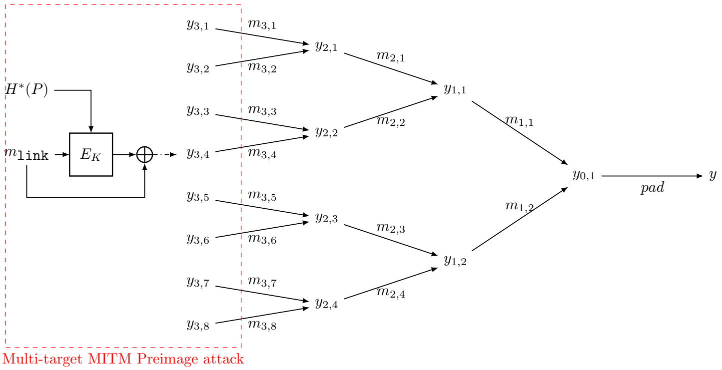
An insight on the herding attack

The online phase of a herding attack can be viewed as a multi-target preimage attack.



The Meet-in-the-Middle Attack

The meet-in-the-middle attack is one of the most effective methods for attacking hash functions.

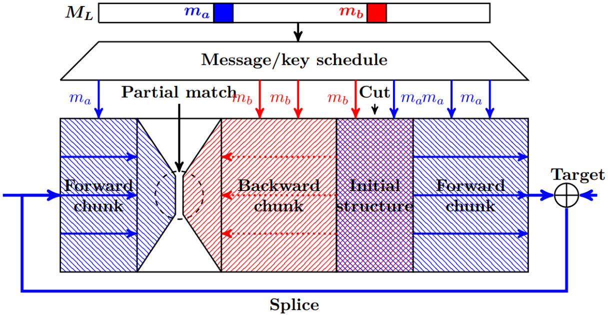


Figure: An overview of the meet-in-the-middle attack.

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The Meet-in-the-Middle Attack

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MILP-based Methods

- First introduced by Bao et al. [Bao+21] at EUROCRYPT 2021.
- Further researches were conducted in [Don+21; SS22; Bao+22; Qin+23].

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Attack Complexity (in log_2)

- Classical : $n - \min(d_{\mathcal{B}}, d_{\mathcal{R}}, d_{\mathcal{M}})$
- Quantum : $(n - \min(|d_{\mathcal{B}} - d_{\mathcal{R}}|, d_{\mathcal{B}}, d_{\mathcal{R}}, d_{\mathcal{M}}))/2$

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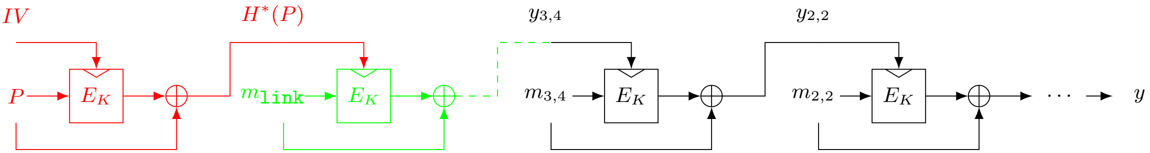
The MITM Nostradamus Attack

The Meet-in-the-Middle Nostradamus Attack

- **Offline phase**: Construct a diamond structure utilizing the previous methods [KK06; BFH22].
- **Online phase**: Mount a meet-in-the-middle attack on the compression function to find a linking message.



Online Phase
MITM Attack

Offline Phase
Collision-based Method

The MITM Nostradamus Attack

Complexities

- Time complexity:

$$\max\left(2^{n-\min(d_{\mathcal{B}},d_{\mathcal{R}},d_{\mathcal{M}})}, \sqrt{k}\cdot 2^{(n+k)/2}\right),$$

- Memory complexity:

$$\max\left(2^{k}, min(2^{d_{\mathcal{B}}}, 2^{d_{\mathcal{R}}})\right).$$

To perform a faster attack than the generic attack, we need (we omit the factor $\sqrt{k}$ here.)

$$k < \frac{n}{3} \text{ and } \min\left(d_{\mathcal{B}}, d_{\mathcal{R}}, d_{\mathcal{M}}\right) > \frac{n}{3}.$$

The Quantum MITM Nostradamus Attack

Complexities

- Time complexity:

$$\max(2^{\frac{1}{2}(n-\min(|d_{\mathcal{B}}-d_{\mathcal{R}}|,d_{\mathcal{B}},d_{\mathcal{R}},d_{\mathcal{M}}))}, \sqrt[3]{k}\cdot 2^{(n+2k)/3}),$$

- Memory complexity:

$$\max\left(2^k, min(2^{d_{\mathcal{B}}}, 2^{d_{\mathcal{R}}})\right).$$

To perform a faster attack than the generic attack, we need (we omit the factor $\sqrt{k}$ here.)

$$k \leq \frac{n}{7}, \min\left(\left|d_{\mathcal{B}} - d_{\mathcal{R}}\right|, d_{\mathcal{B}}, d_{\mathcal{R}}, d_{\mathcal{M}}\right) \geq \frac{n}{7}.$$

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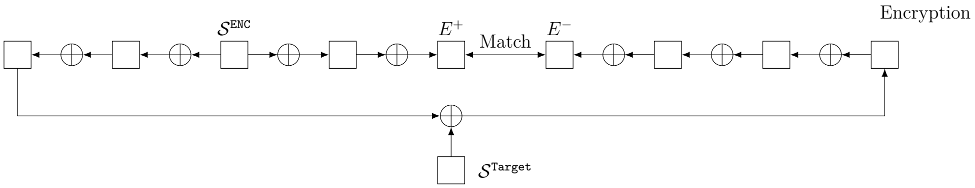
MILP-based search method for MITM Nostradamus Attacks

Based on the MILP model of the MITM preimage attack, we proposed an automated seach method for the MITM Nostradamus attacks on AES-like hashing. The notations and propagation rules are the same as previous works [Bao+21; Don+21; Bao+22].



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MILP-based search method for MITM Nostradamus Attacks

Additional Rules for MITM Nostradamus Attacks

$$\begin{cases} O_{\texttt{mitm}} & \leq & \mathrm{DoF}^{\mathcal{B}}, \\ O_{\texttt{mitm}} & \leq & \mathrm{DoF}^{\mathcal{R}}, \\ O_{\texttt{mitm}} & \leq & \mathrm{DoM}, \end{cases} \quad \begin{cases} O_{\texttt{total}} & \geq & \frac{n+k}{2}, \\ O_{\texttt{total}} & \geq & n - w \cdot O_{\texttt{mitm}}. \end{cases}$$

Additional Rules for Quantum MITM Nostradamus Attacks

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Results

Table: Results of Nostradamus attacks.

Target	Rounds	Time	C-Mem	QRAM	Setting	Ref.
AES-MMO	6/10	$2^{82.7}$	2^{48}	-	Classic	This work
	7/10	2^{56}	-	2^{8}	Quantum	This work
	7/10	$2^{54.1}$	-	2^{14}	Quantum	This work
	any	$2^{88.8}$	$2^{42.6}$	-	Classic	[KK06]
	any	$2^{57.2}$	-	$2^{18.3}$	Quantum	[BFH22]
WHIRLPOOL	4/10	2^{320}	2^{192}	-	Classic	This work
	6/10	$2^{216.7}$	-	2^{64}	Quantum	This work
	any	$2^{351.8}$	$2^{170.6}$	-	Classic	[KK06]
	any	$2^{226.3}$	-	$2^{73.1}$	Quantum	[BFH22]

Conclusions and Future Works

- The first dedicated Nostradamus attack on AES-like hashing.
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Thank you for listening!

Reference I

Some figures are from [Zha+23; KK06; BFH22].

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